Review of Confidence Intervals

- **Confidence interval for the mean; σ is unknown**

$$\bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

where $t_{\alpha/2}$ is a critical value from T-distributionwith $n-1$ degrees of freedom

- **Confidence interval for the difference of means; equal, unknown variances**

$$\bar{X} - \bar{Y} \pm t_{\alpha/2}\, s_p \sqrt{\frac{1}{n} + \frac{1}{m}}$$

where s_p is the pooled standard deviation

$$s_p = \sqrt{s_p^2}; \qquad s_p^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2 + \sum_{i=1}^{m}(Y_i - \bar{Y})^2}{n+m-2} = \frac{(n-1)s_X^2 + (m-1)s_Y^2}{n+m-2}$$

and $t_{\alpha/2}$ is a critical value from T-distribution with $(n+m-2)$ degrees of freedom.

- **Confidence interval for the difference of means; unequal, unknown variances**

$$\bar{X} - \bar{Y} \pm t_{\alpha/2} \sqrt{\frac{s_X^2}{n} + \frac{s_Y^2}{m}}$$

where $t_{\alpha/2}$ is a critical value from T-distribution with k degrees of freedom given by the Satterthwaite approximation

$$k = \frac{\left(\frac{s_X^2}{n} + \frac{s_Y^2}{m}\right)^2}{\frac{s_X^4}{n^2(n-1)} + \frac{s_Y^4}{m^2(m-1)}}$$

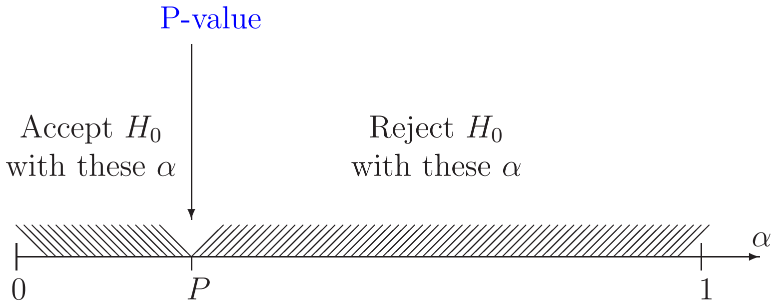
Review of Hypothesis Testing

- **Summary of T-tests**

Hypothesis H_0	Conditions	Test statistic t	Degrees of freedom
$\mu = \mu_0$	Sample size n; unknown σ	$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$	$n-1$
$\mu_X - \mu_Y = D$	Sample sizes n, m; unknown but equal standard deviations, $\sigma_X = \sigma_Y$	$t = \frac{\bar{X} - \bar{Y} - D}{s_p\sqrt{\frac{1}{n} + \frac{1}{m}}}$	$n+m-2$
$\mu_X - \mu_Y = D$	Sample sizes n, m; unknown, unequal standard deviations, $\sigma_X \neq \sigma_Y$	$t = \frac{\bar{X} - \bar{Y} - D}{\sqrt{\frac{s_X^2}{n} + \frac{s_Y^2}{m}}}$	Satterthwaite approximation, see page 1

- **P-value**

  P-value is the lowest significance level α that forces rejection of the null hypothesis. P-value is the highest significance level α that forces acceptance of the null hypothesis. P-value is the probability of observing a test statistic that is as extreme as or more extreme than the test statistic computed from a given sample.



- **Computing P-values for T-tests and Stating Conclusions**

Hypothesis H_0	Alternative H_A	P-value	Computation
$\theta = \theta_0$	right-tail $\theta > \theta_0$	$\mathbf{P}\{t \geq t_{\text{obs}}\}$	$1 - F_k(t_{\text{obs}})$
	left-tail $\theta < \theta_0$	$\mathbf{P}\{t \leq t_{\text{obs}}\}$	$F_k(t_{\text{obs}})$
	two-sided $\theta \neq \theta_0$	$\mathbf{P}\{\lvert t\rvert \geq \lvert t_{\text{obs}}\rvert\}$	$2(1 - F_k(\lvert t_{\text{obs}}\rvert))$

$$\begin{cases} \text{For} \quad \alpha < P, & \text{accept } H_0 \\ \text{For} \quad \alpha > P, & \text{reject } H_0 \\ \textit{Practically,} & \\ \text{If} \quad P < 0.01, & \text{reject } H_0 \\ \text{If} \quad P > 0.1, & \text{accept } H_0 \end{cases}$$

F_k is the cdf of T-distribution with the suitable number k of degrees of freedom

- **Equivalence of two-sided tests and confidence intervals**

  A level α T-test of $H_0 : \theta = \theta_0$ vs $H_A : \theta \neq \theta_0$ accepts H_0 **if and only if** a symmetric $(1-\alpha)100\%$ confidence T-interval for θ contains θ_0.

- **Two-sample F-tests for Variances**

Null Hypothesis $H_0 : \sigma_X^2 = \sigma_Y^2$		Test statistic $F_{\text{obs}} = \dfrac{s_X^2}{s_Y^2}$
Alternative Hypothesis	Rejection region	P-value Use F$(n-1, m-1)$ distribution
$\sigma_X^2 > \sigma_Y^2$	$F_{\text{obs}} \geq F_\alpha(n-1, m-1)$	$\mathbf{P}\{F \geq F_{\text{obs}}\}$
$\sigma_X^2 < \sigma_Y^2$	$F_{\text{obs}} \leq F_\alpha(n-1, m-1)$	$\mathbf{P}\{F \leq F_{\text{obs}}\}$
$\sigma_X^2 \neq \sigma_Y^2$	$F_{\text{obs}} \geq F_{\alpha/2}(n-1, m-1)$ or $F_{\text{obs}} < 1/F_{\alpha/2}(m-1, n-1)$	$2\min\left(\mathbf{P}\{F \geq F_{\text{obs}}\},\ \mathbf{P}\{F \leq F_{\text{obs}}\}\right)$